The Use of Smoothing Splines to Assess Uncertainties in Alpha Curves

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The degree of criticality of a nuclear assembly is quantified by α, the logarithmic derivative of the neutron flux. The evolution of the criticality in time is called the "α-curve." Measurements of the α-curve are one of the most useful sources of information about the performance of a nuclear device.

The measurement of the α-curve is complicated by the extreme physical conditions and short time scales of the nuclear reaction. Ingenious methods of measuring the α-curve have been developed for use in nuclear testing. Chief among these is a technique introduced by the astrophysicist Bruno Rossi which enables the α-curve to be inferred from the trace of a suitably configured oscilloscope. An extraordinary amount of effort has been expended in recording and analyzing Rossi trace data generated by nuclear tests. These measurements, however, have uncertainties, which have not been fully quantified. In order to make full use of the measured data, and to understand which inferences are or are not justified from the data, the uncertainty in the α-curve needs to be quantified. The purpose of the present work is to show how the method of smoothing splines, as developed by Wahba and others [1], can be used to quantify the measurement error in α-curves inferred from Rossi traces. The use of splines to analyze Rossi traces was first introduced by Hanson and Booker [2]. The use of Wahba's formalism was suggested by David Sharp.

The raw data is provided in the form of a Rossi trace, as recorded on photographic film or some similar medium. A synthetic trace is shown in Fig. 1. The film is placed in a reader, and a large number of individual points of the trace are measured. There are two primary sources of error: random measurement uncertainty and systematic distortions due to imperfections in the electronics. Here we restrict our attention to the measurement uncertainty.

The method of smoothing splines can be interpreted in terms of a probabilistic model for noisy measurements of an unknown function f. In our case, $f(t)$ is the signal flux, as a function of time. The prior measure on the space of functions f is chosen to be $(m-1)$-fold integrated Wiener measure [3], which is essentially the m-fold integral of white noise. The likelihood is that corresponding to independent measurements y_i of $f(t_i)$ with variances σ_i^2. With these assumptions, it can be shown that the Bayes' estimate is the solution to the following minimization problem: Choose f to minimize

$$\sum_i w_i \left[y_i - f(t_i) \right]^2 + p \int \left[f^{(m)}(u) \right]^2 du,$$

where w_i are appropriate weights, and p is determined from the data [1]. This is exactly the type of minimization problem that arises in the variational approach to splines, and indeed, the solution is a natural spline of order m, with knots at the measurement points.
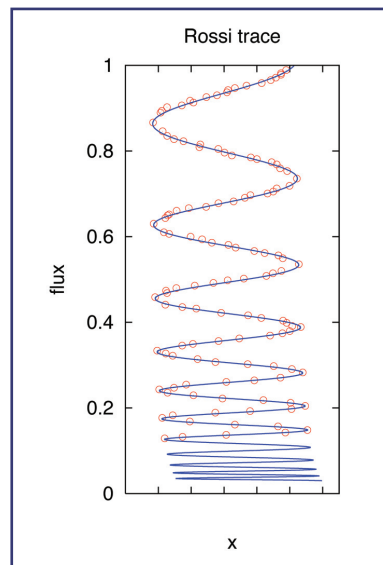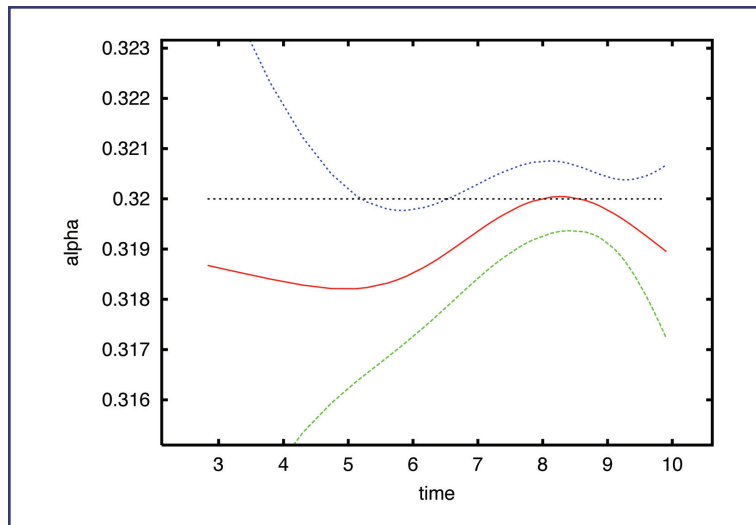

Rossi trace

Figure 1—
A synthetic Rossi trace.

The probabilistic model provides the necessary foundation for estimating *f* as the posterior mean (Bayes' estimate), and also for determining the uncertainty in *f* and its derivatives. In particular, it enables us to compute uncertainties in α, which is the logarithmic derivative of *f*.

Figure 1 shows a synthetically generated Rossi trace, in arbitrary units, with both random and systematic errors. For this data, α is assumed constant, so the flux rises exponentially. The vertical axis is the signal flux. The horizontal trace varies like cos ω*t*, so that *x*(*t*) encodes the time. An initial step, not discussed here, is to recover the time from *x*. The output of this step is a set of noisy flux measurements. The flux curve, the alpha curve, and its uncertainties can be estimated from this data.

The estimate of the α-curve and its uncertainty corresponding to Fig. 1 is shown in Fig. 2. I emphasize that the result shown here should regarded as an example of the technique, and not an assessment of the true uncertainty in experimental α-curves. The latter requires accurate estimates of the uncertainties in the measured data.

[1] Grace Wahba, S*pline Models for Observational Data* **59** in CBMS-NSF Regional Conference Series in Applied Mathematics (SIAM, Philadelphia, 1990).
[2] Kenneth M. Hanson and Jane M. Booker, "Inference from Rossi Traces," in A. Mohammad-Djafari, Ed., *Bayesian Inference and Maximum Entropy Methods in Science and Engineering* **568** (AIP Conf. Proc., Melville, NY, 2001) p. 604.
[3] L. Shepp, "Radon-Nikodym Derivatives of Gaussian Measures," *Ann. Math. Statist.*, **37**, 321–354 (1966).

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